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GROWTH AND YIELD OF SECOND-GROWTH RED GUM IN FULLY STOCKED STANDS ON ALLUVIAL LANDS IN THE SOUTH

By

ROBERT K. WINTERS, *Forester,*

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This paper releases data gathered in current investigations of the Southern Forest Experiment Station, and is subject to correction or modification following further investigation.

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INTRODUCTION

General

The natural range of red gum (*Liquidambar styraciflua* Linn.) includes the eastern United States from Connecticut to the Trinity River, Texas, and the interior as far north as southern Ohio and Indiana. In the maritime region of the South Atlantic and Gulf States and the Mississippi River Delta, it grew to large size in the original timber stands on the better-drained, silty-clay, bottomland soils. Red gum in virgin hardwood forests seldom grew in pure stands over extensive areas, but usually was found in mixture with other species.

Purpose and Scope of Study

On account of the wide distribution, large aggregate volume, and present commercial importance of red gum timber, a study of its growth rate and yield in fully stocked, even-aged, second-growth stands was undertaken by the Southern Forest Experiment Station of the U. S. Forest Service in 1931, the field work of which was completed in 1933. At the outset this study was confined to the Delta region of the Mississippi River; but since second-growth red gum stands are commonly found on the alluvial bottomlands of other southern streams that flow into the Atlantic Ocean and Gulf of Mexico, the scope of the study was extended to these lands. The results therefore apply to second-growth red gum grown in fully stocked stands on alluvial bottomlands throughout the South. Field data were gathered on plots in stands distributed as shown in figure 1.

CHARACTERISTICS OF STANDS STUDIED

Stands in this study were classified as "red gum" if 90 percent or more of the largest and best developed trees were red gum. Although second-growth red gum stands occupy a considerable area in the South, only a small percentage of this area is even-aged and fully stocked. Theoretically, fully stocked stands are those in which the tree crowns form a complete and unbroken canopy. As a rule, however, such stands contain a few small openings. Stands were considered to be even-aged if the age variation among the dominant and codominant trees was not greater than 8 to 10 years.

[1] Acknowledgement is gratefully given Mr. V. B. Davis, Associate Forest Economist, Southern Forest Experiment Station, who was in personal charge of the major part of the field work. Acknowledgement is also due Mr. F. X. Schumacher, Silviculturist, U. S. Forest Service, and Mr. R. A. Chapman, Assistant Silviculturist, Southern Forest Experiment Station, who gave valuable assistance throughout the analysis of the data.

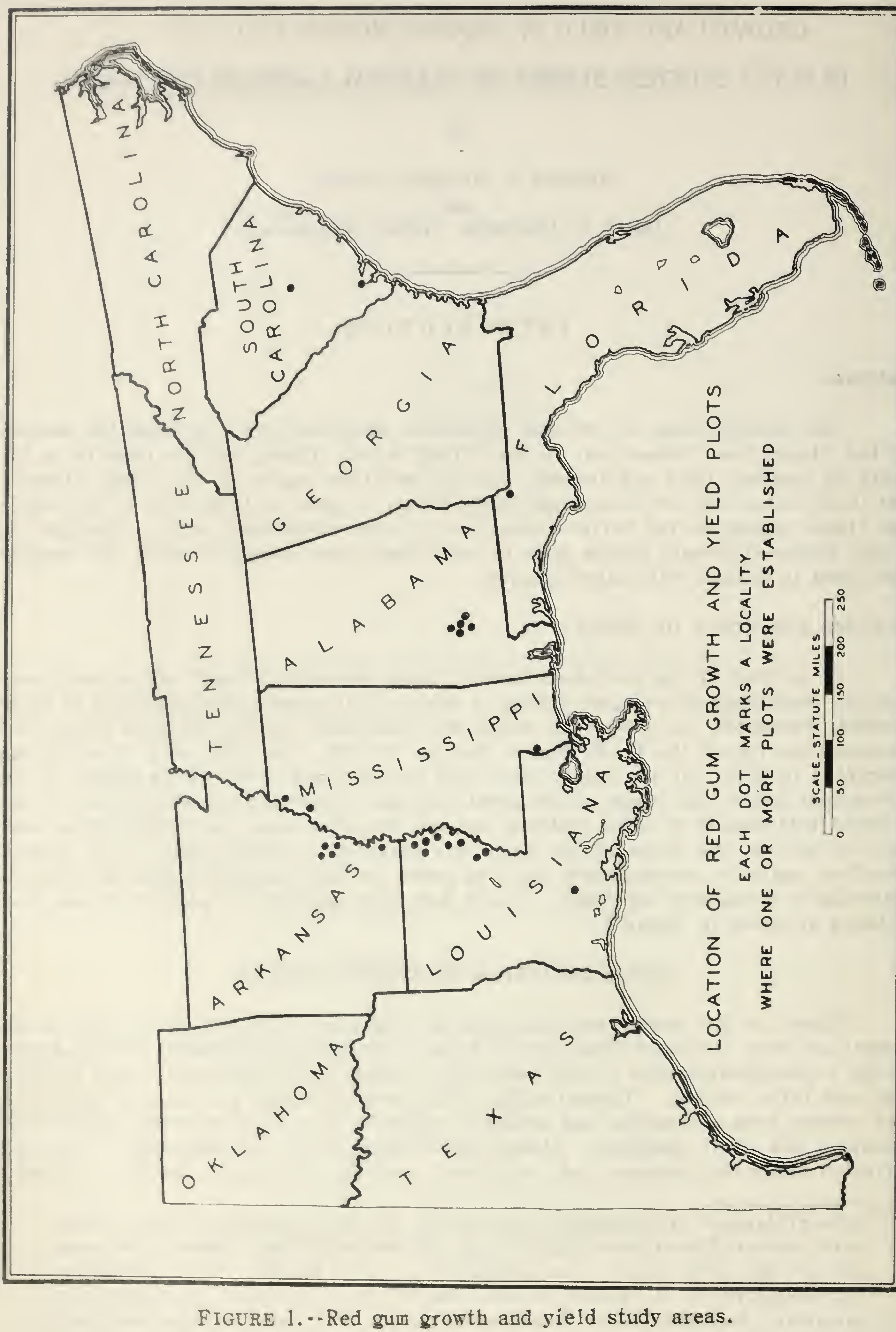

FIGURE 1.--Red gum growth and yield study areas.

The young red gum tree is able to endure a certain amount of shade and stand crowding; hence the leaf and crown canopy of young stands is usually dense. With increased age, however, trees become less able to endure competition. The most suppressed red gum trees die from crowding and the stands become more open. Following this natural decrease in the density of the crown canopy, sufficient sunlight reaches the ground to permit the development of an understory stand, composed of such species as American elm, hackberry, red maple, hawthorn, deciduous holly, and dogwood. This understory is present in nearly all even-aged, second-growth, red gum stands more than 40 years old. (See figure 2.) Since the trees in the understory do not usually contribute to the merchantable volume of the main stand at the utilization date, they are not considered in the timber yield as shown in table 1.

As nearly as could be learned from the analysis of forest conditions on the growth and yield plots, most of the stands had been burned over at some time during the life of the stand. Light ground fires seldom kill a red gum tree after it is 30 or 40 years old, but they do wound the bases of trees and thus facilitate the entrance of decay organisms into the tree stems. Most of the rot found near the base of red gum trees is probably attributable directly or indirectly to fire scarring.

Forest fires of moderate intensity frequently kill nearly all the trees in stands less than 10 to 15 years old. At this age red gum sprouts vigorously, however, and the stand that subsequently develops may be either partly or entirely of sprout origin, depending upon the completeness with which the original seedling stand was destroyed by fire. The early height growth of the red gum sprouts from a vigorous root system is exceedingly rapid. Seedlings require, on the average, 3 to 5 years to reach breast height, whereas sprouts from vigorous root stocks frequently reach breast height at the end of the first growing season. Although the duration of this accelerated growth has not been specifically determined, 10-year-old red gum sprouts frequently have the size and general appearance of 18-to 20-year-old seedlings in the same stand. Red gum stands of mixed sprout and seedling origin are readily identified by the presence of "twins", that is, two or more trees that have sprouted from a single root stock. (See figures 3 and 4.)

DEFINITION OF TERMS

Diameter at breast height.--The outside-bark diameter of a standing tree measured 4.5 feet above the ground. This term is commonly abbreviated to d.b.h.

Number of trees.--All trees above a minimum stated d.b.h. that are considered to be a part of the even-aged stand. This concept of number of trees excludes trees of species that generally form the understory characteristic of even-aged red gum stands more than 40 years old.

Basal area.--The area in square feet of a cross-section of a tree measured at breast height. The basal area of a stand is the sum of the basal areas of the trees of which it is composed.

Average diameter at breast height.--The diameter of a tree that has the average basal area of the trees in the stand. The calculation of the average diameter at breast height involves two separate operations: (*a*) Dividing the basal area of a stand by the number of trees it contains to obtain the average-tree basal area; and (*b*) Determining, by reference to standard tables, the tree diameter in inches that corresponds to this average basal area.

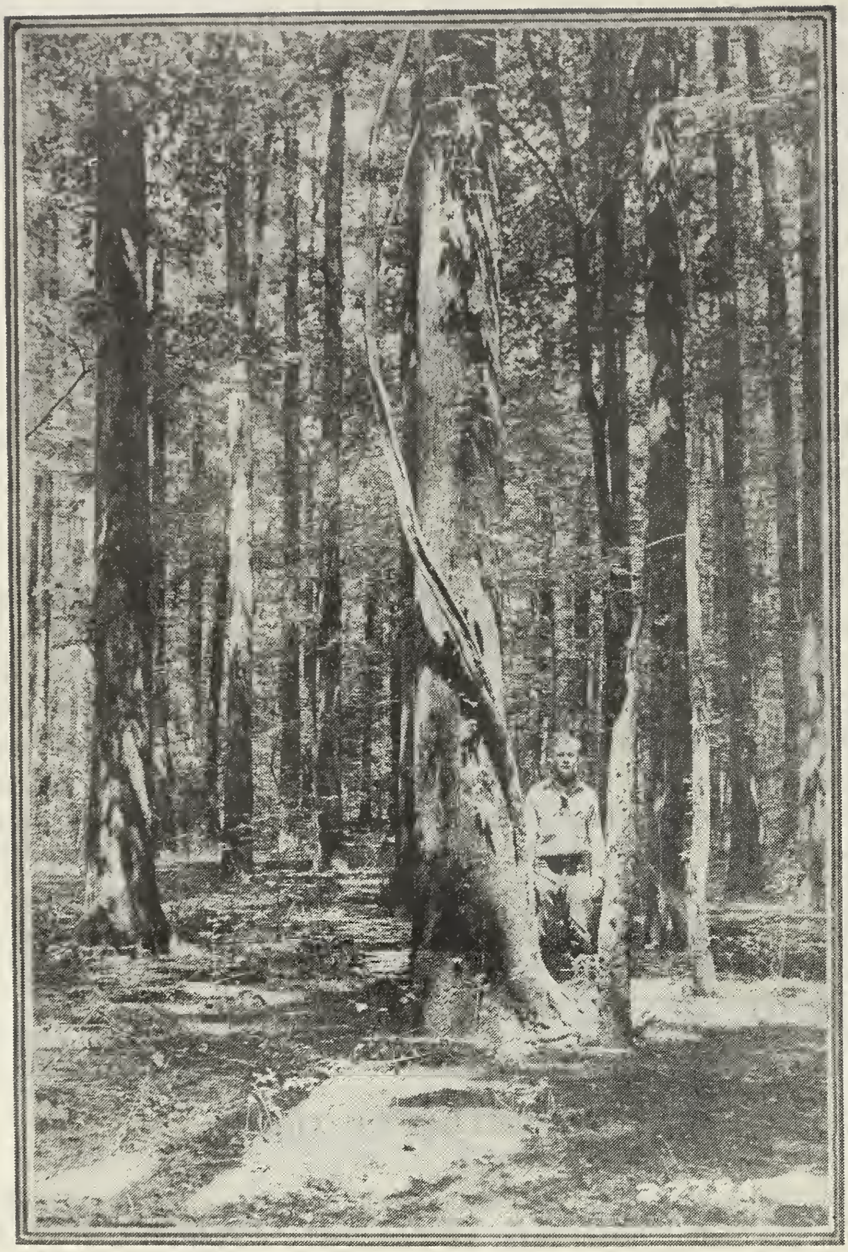

FIGURE 2.--A 67-year-old red gum stand in southern Mississippi. Note the conspicuous understory that has developed beneath the maturing stand.

FIGURE 3.--An even-aged, second-growth stand of young red gum. This 14-year-old stand in northern Louisiana developed on an abandoned cotton field. Note the "twin" trees that indicate the sprout origin of at least a part of the stand.

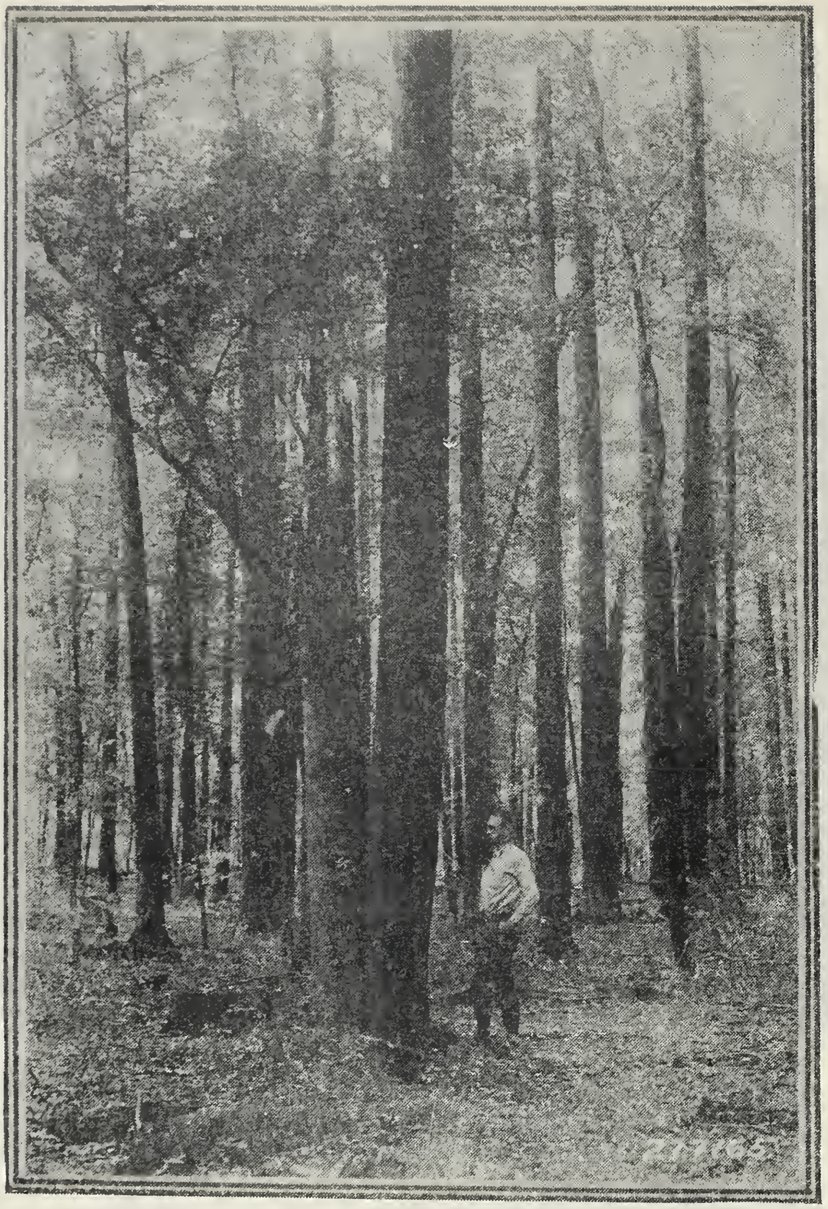

FIGURE 4.--A 99-year-old second-growth red gum stand in southeastern Alabama. Note that the tree in the foreground developed as a "twin".

Dominant.--Any tree with a well formed crown that receives full sunlight from above and at least some light from the side. The dominant classification, as used in this study, includes both the dominant and codominant trees, as ordinarily defined.

Stand age.--The average age at breast height of 5 to 10 dominant trees considered to be representative of the stand.

Site quality.--The relative wood-producing capacity of a given area, measured in this study by the height growth of the average dominant red gum.

Site index.--A measure of site quality. In this study site index is the height attained by the average dominant tree 50 years of age at breast height.

Merchantable tree.--Any tree over 12.5 inches in diameter at breast height that contains one merchantable log (see below) at least 12 feet long.

APPLICATION OF GROWTH AND YIELD DATA

The growth and yield figures that follow are intended for use in calculating the growth and yield of even-aged, fully stocked, second-growth red gum stands found on the alluvial bottomlands of the South. They cannot properly be applied to stands that do not meet these specifications. Since stands can deviate from these so-called "normal" stands in a number of ways, no specific corrections can be given to make these tables apply to partially stocked stands or to uneven-aged stands or to stands of mixed species. Skilled men, who know red gum timber stands and the laws of forest growth, can sometimes make approximate predictions for "unnormal" stands that do not deviate too greatly from the normal.

MERCHANTABLE STAND

Description of log grades

Log grades used by the Southern hardwood industry vary greatly throughout the region, depending upon locality and fluctuations in the hardwood lumber market. The following grades for merchantable logs, developed by the Southern Forest Survey, have been used in this study:

Grade 1.--A sound log [2] at least 12 feet long and 16 inches in diameter inside bark at the small end, from which at least 70 percent [3] of the volume can be cut into number 1 common lumber or better (either sap gum or red gum).

Grade 2.--A sound log at least 12 feet long and 12 inches in diameter inside bark at the small end, from which 35 to 69 percent [3] of the volume can be cut into number 1 common lumber or better (either sap gum or red gum).

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[2] A sound log is one that has at least 50 percent of its volume in sound material.

[3] These percentages are the minimum for the given grade. A run of number 1 logs, for example, will yield more than 70 percent of their aggregate volume in number 1 common lumber or better.

Grade 3.--A sound log at least 12 feet long and 9 inches in diameter inside bark at the small end, which cannot be classified as grade 2 because of either size or sweep. A log of this grade is especially suitable for the production of barrel staves and small dimension stock.

Grade 4.--A sound log at least 12 feet long and 10 inches[4] in diameter inside bark at the small end, which cannot be classified as grade 3 or better because of size or quality. At least 50 percent of its volume must be suitable for ties, timbers, or rough structural material. Logs of this grade are usually rough and knotty, and may be of any diameter above the minimum specified.

Utilization Practices

Although logs that meet the requirements of the preceding specifications are considered merchantable throughout this report, the average hardwood-lumber mill operator would not use all of the logs in all of the log grades. Casual inspection of cutting operations in fully stocked, even-aged, second-growth red gum stands indicates that a considerable quantity of material classified as log grade 4 is commonly left in the woods after a lumber-mill or barrel-stave cutting operation.[5]

Field Measurements

On each sample plot established in this study, the breast-high diameter of each tree was measured with calipers. Ocular estimates were made also of the length and of the inside-bark diameter at the top end of each merchantable log in each tree. Each log was graded in accordance with the log-grade definitions given above, the merchantable length of each tree being divided into 12-, 14-, and 16-foot logs in such a way as to obtain a minimum volume in low-grade logs. The merchantable length of a given tree depended upon the limbiness or knottiness of the upper part of the stem, and incuded only such part as could be graded log grade 4 or better. Although the diameter inside bark at the top of the highest log varied from tree to tree, it was seldom less than 10 inches. An occasional 9 inch log was included in the volume of log grades 3 and 4.

In estimating board foot volumes of individual trees, that quantity of defective stem material was excluded which, according to generally accepted logging practice, would be left unutilized in the woods. Accordingly, the volume of cull sections was

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[4] An occasional 9-inch log of better-than-average quality was included in this grade.

[5] The following data were obtained on a selected acre plot in a fully stocked, even-aged, second-growth red gum stand. The timber was cut for lumber-mill utilization and the data are presented here for what they may be worth. Woods-utilization practice on this area is thought to be somewhat closer than is usual in similar stands of red gum elsewhere in the South.

Age of stand at breast height	67	years
Average total height of dominant trees in stand	111	feet
Volume in standing trees before logging	14,189	bd. ft. per acre (Scribner log scale)
Volume in trees cut	6,890	bd. ft.
Percentage of total volume felled	49	percent
Percentage of felled material hauled to mill	73	percent
Percentage of felled material left in woods (including volume of wood split or otherwise damaged in felling)	27	percent

deducted from all badly fire-scarred trees and from partially rotten, crooked, or forked trees. The tables of merchantable volume show the gross contents of merchantable logs, and include the cull that is ordinarily carried to the mill in the log. Experience indicates that this cull amounts to approximately 1 percent in the stands studied.

Merchantable Yield and Volume Tables

In the analysis of the field data taken on each plot bearing trees of merchantable size, both the Doyle and the Scribner log rules were applied to the logs by log grades in order to arrive at the plot volume classified by log grades. These plot volumes were weighted and curved to give the average merchantable board-foot yield per acre in stands of different ages and sites. These volumes, together with Doyle and Scribner tree-volume tables and other pertinent data, are shown in tables 1 to 3.

The Use of the Merchantable Yield Tables

These tables are useful chiefly for predicting timber yields by log grades in fully stocked, even-aged, red gum stands. To use these tables for this purpose, it is first necessary to determine the age and average height of dominant trees, and the degree of stocking of the stands for which such forecasts are to be made. Average stand age can be determined by counting the annual rings on cores of wood extracted at breast height with an increment borer[6] from the largest and best-developed trees in the stand. The height of these trees should be carefully measured with a Forest Service Abney level[7] or some other instrument of equal accuracy.

Whenever increment borers are not available, approximate age determinations can be made by counting the annual growth rings on freshly cut stumps. One year can be subtracted from stump-height age to get the approximate breast-height age. In the absence of an Abney level or similar instrument, total tree heights can be roughly determined by ocular estimate or preferably by measuring the height of felled trees.

Having determined the age and height of the dominant trees, one may classify the area on which the stand is growing as a good, medium, or poor site. For example, if the height of the dominant trees in a given 60-year-old stand was found to be approximately 108 feet, the site would be of medium quality (see first and second columns of table 1). To ascertain whether the given stand is fully stocked, count the number of merchantable trees (trees more than 12.5 inches in diameter) on a representative acre, and compare that number with the figure given in column 3 of table 1 opposite the 60 in column 1 of the medium-site section. If the number is approximately 76, the stand is fully stocked and the yield tables can be safely applied.

The average acre of this 60-year-old stand growing on a medium site should bear 9,880 board feet (Doyle log rule) of timber of all log grades. Of this volume, 840 board feet per acre should be grade 1; 2,540, grade 2; 2,600, grade 3; and 3,900, grade 4. This statement does not mean that any acre chosen at random from a 60-year-old, fully stocked, second-growth stand can be expected to have exactly this volume divided among grades 1, 2, 3, and 4. The average of a number of such acres should, however, have approximately this total volume, and it should be divided among grades 1, 2, 3, and 4 in approximately the manner indicated in the tables.

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[6] An increment borer is a small (8"-12") auger-like instrument that, by being bored into a tree, can remove a "pencil" of wood. On this core, the rings of growth can be seen, counted, and measured to determine the rate of tree-diameter growth.

[7] A small hand level that can be used to measure tree height or percent of slope.

TABLE 1. -- *Merchantable board-foot volume*[1] *per acre, classified by site, stand-age, and log grades (Scribner and Doyle log rules)*

Age at breast-height	Average height of dominant trees	Merchantable trees	Average breast-high diameter of merchantable trees	Basal area of merchantable trees	Volume per acre: Scribner Log Rule, Grade 1	Scribner Grade 2	Scribner Grade 3	Scribner Grade 4	Scribner Total	Doyle Log Rule, Grade 1	Doyle Grade 2	Doyle Grade 3	Doyle Grade 4	Doyle Total
Years	Feet	Number	Inches	Sq. ft.	Bd. ft.	Bd. ft.	Bd. ft.	Bd. ft.	Bd. ft.	Bd. ft.	Bd. ft.	Bd. ft.	Bd. ft.	Bd. ft.
POOR SITE - (SITE INDEX 80)														
40	69	7	14.6	7			70	330	400			20	230	250
50	80	30	14.8	33		60	740	1,200	2,000			520	850	1,370
60	88	49	15.1	57	40	540	1,460	2,030	4,070		500	1,080	1,480	3,060
70	94	62	15.4	75	220	1,240	2,020	2,800	6,280	170	1,080	1,520	2,110	4,880
80	99	71	15.9	88	480	1,960	2,460	3,530	8,430	460	1,640	1,860	2,680	6,640
90	104	77	16.5	98	840	2,700	2,800	4,200	10,540	800	2,260	2,080	3,200	8,340
MEDIUM SITE - (SITE INDEX 100)														
40	89	44	14.8	58		370	1,700	2,190	4,260		310	1,080	1,550	2,940
50	100	64	15.4	91	260	1,510	2,720	3,690	8,180	220	1,390	1,960	2,750	6,320
60	108	76	16.1	115	880	3,040	3,450	5,030	12,400	840	2,540	2,600	3,900	9,880
70	115	86	17.0	133	1,750	4,530	3,970	6,260	16,510	1,630	3,840	3,060	4,960	13,490
80	122	93	17.8	146	2,720	5,930	4,370	7,440	20,460	2,490	4,980	3,420	5,970	16,860
90	127	97	18.6	156	3,700	7,240	4,680	8,500	24,120	3,340	5,960	3,700	6,920	19,920
100	131	99	19.3	163	4,610	8,390	4,940	9,420	27,360	4,180	6,860	3,930	7,820	22,790
GOOD SITE - (SITE INDEX 120)														
30	93	42	14.7	47		110	1,200	2,120	3,430			880	1,460	2,340
40	108	75	15.4	101	250	1,870	3,230	4,500	9,850	200	1,570	2,400	3,300	7,470
50	120	89	16.3	136	1,320	4,160	4,530	6,700	16,710	1,220	3,580	3,450	5,200	13,450
60	129	97	17.4	161	2,800	6,700	5,350	8,770	23,620	2,580	5,620	4,140	6,950	19,290
70	137	103	18.5	179	4,500	9,030	5,950	10,640	30,120	4,080	7,460	4,660	8,680	24,880
80	144	107	19.6	190	6,180	11,000	6,430	12,330	35,940	5,540	8,930	5,100	10,240	29,810

[1] Including interior rot and other hidden cull. Sawmill experience indicates that this amounts to approximately 1% of the merchantable volume as shown in the tables.

Should the owner of this 60-year-old stand wish to harvest his timber at 80 years of age instead of 60, he could expect to secure on the average 16,860 board feet per acre (Doyle log rule). He could reasonably expect 2,490 board feet per acre of this volume to be in grade 1 logs; 4,980 in grade 2; 3,420 in grade 3; and 5,970 in grade 4. Knowing the present average selling price of lumber or other products from logs of the several grades, he can obtain from table 1 the stand-growth information needed in deciding when he should cut his stand.

TABLE 2.--*Board-foot volume table, merchantable volume of second-growth red gum (Doyle rule - merchantable height)*

D.B.H. (inches)	Volume (board feet) by 16-foot log lengths					Basis (trees)
	1	2	3	4	5	
13	33	72	119			287
14	40	86	140			295
15	47	99	159	224		248
16	56	114	179	249	330	236
17	65	130	201	270	370	170
18	74	149	230	319	418	103
19	88	170	256	350	452	88
20	99	193	290	394	510	74
21		215	320	433	560	47
22		246	360	483	620	30
23		284	410	545	700	23
24		320	451	593	750	14
25		362	510	665	835	11
Total						1,626

Constructed by R. A. Chapman by curving merchantable volume over merchantable length by one-inch diameter classes. Stump height 2 feet. Volume based on estimated diameter inside bark at top of logs in standing trees. Log diameter at merchantable limit variable; in general, minimum was 9.6 inches. In the aggregate, tabular volumes are 0.5 percent lower than the basic data. A 3-tenths of a foot trimming allowance was made on each log. Block indicates extent of basic data.

TABLE 3.--*Board-foot volume table, merchantable volume of second-growth red gum (Scribner rule - merchantable height)*

D.B.H. (inches)	Volume (board feet) by 16-foot log lengths					Basis (trees)
	1	2	3	4	5	
13	46	98	156			288
14	51	110	176			299
15	59	126	196	274		253
16	66	140	220	303	403	233
17	77	158	244	332	435	171
18	87	176	271	371	482	100
19	98	199	302	410	532	87
20	115	230	343	461	598	75
21		262	384	510	648	46
22		298	431	568	710	30
23		325	469	610	768	23
24		355	505	650	808	14
25		385	540	678	848	10
Total						1,629

Constructed by R. A. Chapman by curving merchantable volume over merchantable length by one-inch diameter classes. Stump height 2 feet. Volume based on estimated diameter inside bark at top of logs in standing trees. Log diameter at merchantable limit variable; in general, minimum was 9.6 inches. In the aggregate, tabular volumes are 0.5 percent lower than the basic data. A 3-tenths of a foot trimming allowance was made on each log. Block indicates extent of basic data.

APPENDIX

A

TOTAL AND PARTIAL STAND, VOLUME, AND YIELD TABLES.

TABLE 4. -- *Height of average dominant tree*

Age at b.h. (years)	Height (feet) by site index					
	70	80	90	100	110	120
10		17	22	30	39	47
20	31	40	48	57	66	75
30	47	56	66	75	84	93
40	60	69	79	89	98	108
50	70	80	90	100	110	120
60	77	88	98	108	119	129
70		94	105	115	126	137
80		99	110	122	133	144
90		104	115	127	139	
100			119	131	143	

TABLE 5. -- *Number of trees per acre 0.6 inch d.b.h. and larger*

Age at b.h. (years)	Number of trees by site index					
	70	80	90	100	110	120
10		11,900	8,120	5,850	5,120	4,700
20	4,600	3,130	2,150	1,610	1,400	1,290
30	1,760	1,210	865	651	575	530
40	860	598	431	337	303	284
50	512	372	281	225	205	194
60	358	266	209	174	162	155
70		213	174	150	141	136
80		182	154	135	130	126
90		161	140	126	121	
100			129	118	115	

TABLE 6. -- *Average diameter of trees more than 0.6 inch d.b.h., based on diameter breast high of tree of average basal area*

Age at b.h. (years)	Average diameter breast high (inches) by site index					
	70	80	90	100	110	120
10		1.1	1.6	2.0	2.3	2.6
20	1.6	3.0	3.8	4.6	5.1	5.6
30	3.1	4.9	6.2	7.0	7.8	8.4
40	4.6	6.7	8.2	9.2	10.2	11.0
50	6.0	8.4	10.0	11.2	12.3	13.2
60	7.3	10.0	11.6	12.9	14.0	15.0
70		11.3	13.0	14.4	15.6	16.6
80		12.4	14.2	15.6	16.8	17.9
90		13.4	15.3	16.7	18.0	
100			16.2	17.7	19.0	

TABLE 7. -- *Basal area per acre in trees 0.6 inch d.b.h. and larger*

Age at b.h. (years)	Basal area (square feet) by site index					
	70	80	90	100	110	120
10		77	88	99	107	116
20	91	115	132	147	159	172
30	99	126	145	160	174	188
40	102	129	148	164	178	192
50	103	131	150	166	180	195
60	105	132	152	168	183	197
70		134	154	170	185	199
80		135	155	172	186	201
90		136	156	173	187	
100			157	174	189	

TABLE 8. -- *Cubic-foot volume*[1] *per acre of all trees 0.6 inch d.b.h., inside bark, and larger*

(*PEELED WOOD*)

Age at b.h. (years)	Volume (cubic feet) by site index					
	70	80	90	100	110	120
10		450	690	980	1,300	1,620
20	1,100	1,560	2,110	2,750	3,450	4,150
30	1,730	2,370	3,120	3,950	4,900	5,850
40	2,210	2,960	3,860	4,870	5,990	7,130
50	2,600	3,470	4,480	5,660	6,940	8,240
60	2,950	3,900	5,040	6,350	7,790	9,220
70		4,290	5,530	6,950	8,500	10,090
80		4,640	5,960	7,470	9,140	10,830
90		4,980	6,370	7,950	9,700	
100			6,750	8,400	10,220	

[1] This is the inside-bark volume of the main stem, excluding limbs.

TABLE 9. -- *Check of basic data against yield tables for stand 0.6 inch d.b.h. and larger*

Item	Aggregate deviation[1]	Average deviation[2]	Correlation index[3]
	----------- *Percent* -----------		
Basal area per acre	- .076	11.3	.716
Number of trees per acre	- 1.72	24.7	.917
Average diameter at breast height	+ .79	10.8	.965
Volume in cubic feet	- 1.23	11.3	.954

1/ Aggregate deviation in percent is computed from

$$\frac{\text{Sum (Actual)} - \text{Sum (Estimate)}}{\text{Sum (Estimate)}} \times 100.$$

2/ Average percentages deviation is computed from

$$\frac{\text{Sum}\left[\frac{(\text{Actual} - \text{Estimate})}{\text{Estimate}}\right] \times 100.}{\text{Number of plots}}$$

3/ Correlation index = $\sqrt{1 - \left(\frac{\sigma \text{ est.}}{\sigma y}\right)^2}$

Where σ est. = The standard error of estimate.
σ y = Standard deviation of the dependent variable.

TABLE 10. -- *Number of trees per acre 4.6 inches d.b.h. and larger*

Age at b.h. (years)	Number of trees by site index					
	70	80	90	100	110	120
10						190
20		242	549	627	687	738
30	12	540	553	479	467	456
40	336	425	362	307	285	270
50	316	321	260	212	198	189
60	275	249	203	170	160	154
70		205	170	149	141	136
80		176	151	134	128	125
90		157	138	125	121	
100			128	118	115	

TABLE 11. -- *Average diameter of trees 4.6 inches d.b.h. and larger*

Age at b.h. (years)	Average diameter breast high (inches) by site index					
	70	80	90	100	110	120
10						5.5
20		5.6	5.9	6.2	6.5	6.7
30	5.5	6.4	7.1	7.8	8.3	8.8
40	6.2	7.5	8.6	9.6	10.4	11.1
50	7.0	8.9	10.2	11.4	12.4	13.3
60	8.0	10.2	11.8	13.0	14.1	15.1
70		11.5	13.2	14.5	15.7	16.6
80		12.6	14.4	15.8	17.0	17.9
90		13.6	15.4	16.9	18.1	
100			16.3	17.8	19.1	

TABLE 12. -- *Basal area per acre in trees 4.6 inches d.b.h. and larger*

Age at b.h. (years)	Basal area (square feet) by site index					
	70	80	90	100	110	120
10						17
20		32	79	109	129	148
30	4	99	130	150	167	183
40	75	119	143	162	176	192
50	92	127	149	166	180	195
60	100	131	152	168	182	198
70		134	154	170	184	200
80		135	155	172	186	201
90		136	156	173	188	
100			157	174	189	

TABLE 13. -- *Cubic-foot volume*[1] *per acre of all trees 4.6 inches d.b.h. and larger*

(*PEELED WOOD*)

Age at b.h. (years)	Volume (cubic feet) by site index					
	70	80	90	100	110	120
10						180
20		350	850	1,500	2,150	2,850
30	550	1,450	2,300	3,150	4,100	5,050
40	1,400	2,300	3,350	4,300	5,450	6,550
50	1,950	3,000	4,050	5,250	6,500	7,800
60	2,450	3,550	4,650	6,000	7,400	8,800
70		4,000	5,200	6,650	8,100	9,650
80		4,350	5,650	7,200	8,800	10,400
90		4,700	6,050	7,650	9,350	
100			6,450	8,100	9,950	

[1] This is the inside-bark volume of the main stem to a 4 inch top, inside bark, and excludes limbs.

TABLE 14. -- *Number of trees per acre 6.6 inches d.b.h. and larger*

Age at b.h. (years)	Number of trees by site index					
	70	80	90	100	110	120
20			103	180	279	355
30		195	301	300	321	335
40	101	258	260	240	238	237
50	167	235	214	191	184	181
60	183	203	181	162	154	150
70		180	162	144	138	134
80		164	147	131	127	124
90		151	136	123	119	
100			126	116	114	

TABLE 15. -- *Average diameter of trees 6.6 inches d.b.h. and larger*

Age at b.h. (years)	Average diameter breast high (inches) by site index					
	70	80	90	100	110	120
20			7.4	7.6	7.9	8.2
30		7.8	8.4	9.0	9.5	9.9
40	7.6	8.8	9.6	10.5	11.2	11.8
50	8.4	9.9	11.0	12.0	12.8	13.5
60	9.2	11.0	12.3	13.4	14.3	15.2
70		12.0	13.5	14.7	15.7	16.8
80		12.9	14.6	15.9	17.0	18.1
90		13.8	15.6	17.0	18.2	
100			16.4	17.9	19.2	

TABLE 16. -- *Basal area per acre in trees 6.6 inches d.b.h. and larger*

Age at b.h. (years)	Basal area (square feet) by site index					
	70	80	90	100	110	120
20			23	48	73	97
30		51	95	122	144	164
40	33	94	127	150	168	186
50	65	114	141	162	178	194
60	83	124	148	167	182	197
70		130	152	170	184	199
80		134	154	172	186	201
90		136	156	173	187	
100			156	173	188	

TABLE 17. -- *Board-foot volume per acre*[1] *to 5 inch top diameter inside bark, International rule, 1/8 inch saw kerf*

Age at b.h. (years)	Volume (board feet) by site index					
	70	80	90	100	110	120
20			1,000	3,100	5,800	8,800
30		3,400	8,300	12,900	18,700	24,400
40	2,600	9,200	15,600	22,300	30,000	37,500
50	6,500	14,600	22,200	30,300	39,200	48,200
60	10,400	19,300	27,500	36,700	46,900	56,800
70		23,100	31,900	41,900	53,000	64,000
80		26,300	35,900	46,500	58,200	70,000
90		29,300	39,400	50,700	62,700	
100			42,800	54,500	66,800	

[1] This includes all trees 6.6 inches d.b.h. and larger having at least one log 12 feet long, and 5 inches in diameter inside bark.

TABLE 18. -- *Number of trees per acre 9.6 inches d.b.h. and larger*

Age at b.h. (years)	Number of trees by site index					
	70	80	90	100	110	120
20					21	39
30		12	52	84	117	148
40	4	65	107	126	148	165
50	24	104	133	137	146	151
60	57	124	137	132	133	134
70		132	134	126	124	124
80		132	128	119	118	117
90		127	122	114	112	
100			116	109	108	

TABLE 19. -- *Average diameter of trees 9.6 inches d.b.h. and larger*

Age at b.h. (years)	Average diameter breast high (inches) by site index					
	70	80	90	100	110	120
20					10.7	10.9
30		10.7	11.0	11.3	11.6	11.9
40	10.8	11.2	11.7	12.2	12.7	13.1
50	11.0	11.8	12.5	13.1	13.9	14.5
60	11.4	12.5	13.4	14.2	15.1	15.9
70		13.2	14.3	15.3	16.3	17.3
80		13.9	15.2	16.5	17.5	18.5
90		14.6	16.0	17.5	18.6	
100			16.8	18.4	19.6	

TABLE 20. -- *Basal area per acre in trees 9.6 inches d.b.h. and larger*

Age at b.h. (years)	Basal area (square feet) by site index					
	70	80	90	100	110	120
20					9	17
30		6	26	53	83	107
40	3	37	77	109	136	158
50	16	75	112	140	162	182
60	42	99	132	156	174	192
70		113	142	164	180	196
80		122	149	169	184	198
90		128	152	171	185	
100			154	171	186	

TABLE 21. -- *Board-foot volume per acre*[1] *to 8 inch top diameter inside bark, Scribner rule*

Age at b.h. (years)	Volume (board feet) by site index					
	70	80	90	100	110	120
20					100	1,000
30		200	1,500	3,800	7,000	11,000
40	200	2,200	6,500	10,900	16,100	22,100
50	1,100	6,400	12,000	18,000	24,700	32,100
60	3,400	10,300	16,700	24,000	32,400	41,600
70		13,800	21,100	29,700	39,700	50,300
80		16,900	25,300	35,100	46,200	57,800
90		19,800	29,200	40,000	51,900	
100			32,700	44,300	56,500	

[1] This includes all trees 9.6 inches d.b.h. and larger having at least one log 12 feet long, and 8 inches in diameter inside bark.

TABLE 22. -- *Percentage distribution of trees in and above successive diameter classes in stands of different average diameter*

Diameter class (inches)	Average diameter of stand in inches														
	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
2	58.6	84.7	94.3	98.1	99.1	99.7	99.9	99.9	99.9	99.9	99.9	99.9	99.9	99.9	99.9
3	21.0	54.1	76.8	91.7	96.0	97.5	98.5	98.8	98.9	99.0	99.1	99.2	99.3	99.4	99.5
4	4.0	22.3	49.2	72.2	86.5	92.9	95.5	96.4	97.0	97.4	97.7	98.0	98.3	98.6	98.8
5	.3	8.2	25.7	47.0	68.9	82.1	88.3	92.4	93.9	95.0	95.6	96.6	97.2	97.7	98.0
6		3.5	12.3	27.5	47.1	65.9	77.6	84.8	88.3	91.1	92.7	94.6	95.9	96.7	97.0
7		1.1	6.3	15.1	29.1	46.9	63.0	73.9	80.4	85.1	88.7	92.0	94.1	95.4	95.9
8		.1	3.1	8.3	17.1	31.8	47.0	60.4	70.1	77.4	83.6	88.8	91.8	93.6	94.3
9			1.3	4.3	9.7	20.0	33.8	47.1	58.9	68.3	77.0	84.0	88.5	91.0	92.1
10			.3	2.0	5.5	12.0	23.0	35.0	47.0	58.1	68.9	77.7	83.9	87.5	89.4
11				.7	2.9	7.0	14.7	24.7	36.1	47.9	59.4	69.6	77.6	82.5	85.7
12				.1	1.4	3.8	8.7	16.4	26.9	38.0	49.1	60.0	69.4	76.5	80.5
13					.5	1.9	5.0	10.2	19.1	28.8	38.9	49.3	60.0	69.0	74.5
14						.7	2.6	6.1	13.0	21.0	29.2	38.6	49.1	60.1	67.1
15							1.2	3.7	8.3	14.7	21.2	29.0	38.6	49.8	59.0
16							.4	2.0	5.2	9.6	14.7	20.8	29.1	39.3	50.0
17								.9	3.1	6.2	9.6	14.1	21.2	30.0	41.0
18								.3	1.8	4.0	6.1	9.1	15.0	22.0	32.3
19									.9	2.3	3.8	6.0	10.0	15.6	24.7
20									.3	1.1	2.0	3.6	6.7	10.5	18.1
21										.4	1.0	2.0	4.4	7.1	12.9
22											.3	1.0	2.8	4.9	8.8
23												.4	1.7	3.1	5.9
24												.1	1.0	2.0	3.9
25													.5	1.1	2.4
26													.2	.4	1.6
27															1.0
28															.5
29															.1

TABLE 23. -- *Total volume in cubic feet**

(*PEELED WOOD*)

Diameter breast high	Total height of trees in feet													Basis (trees)
	10	20	30	40	50	60	70	80	90	100	110	120	130	
1	0.05	0.07	0.09											2
2	0.07	0.15	0.24	0.33										4
3	0.11	0.28	0.47	0.68	0.89									5
4	0.15	0.45	0.78	0.15	1.52	1.90								7
5		0.65	1.18	1.72	2.30	2.91	3.50							7
6		0.91	1.61	2.41	3.24	4.10	5.02	5.85						9
7				3.18	4.33	5.48	6.72	7.95	9.13					12
8				4.03	5.49	7.10	8.67	10.2	11.7					5
9					6.85	8.90	10.8	12.7	14.7	16.7				10
10						10.7	13.0	15.5	18.0	20.6	22.9			8
11						12.8	15.6	18.5	21.7	24.9	27.5			10
12						14.9	18.3	21.9	25.5	29.2	32.5			15
13							21.3	25.6	29.9	34.0	38.3	42.2		10
14							24.5	29.5	34.1	39.0	44.0	48.5		9
15							27.9	33.5	38.8	44.3	49.9	55.7		12
16								37.5	44.1	50.3	56.8	63.1		9
17								42.0	48.9	56.4	63.4	71.0		8
18									54.5	62.8	70.9	78.8		10
19									60.8	69.9	78.3	87.5		7
20									66.9	76.0	87.0	96.1		3
21									73.0	83.7	95.0	106		5
22									79.8	92.5	102	116		2
23									87.0	100	113	126	140	3
24									94.5	107	121	135	149	1
25										117	131	147	160	3
26										125	141	158	172	2
27										135	151	168	186	2
Basis (trees)		2	4	13	11	15	16	21	30	34	30	3	1	180

* Volume inside bark of entire stem excluding limbs.

Block indicates extent of basic data.

Values computed from the equation: $\log(V-.04) = 1.892637 \log D + 1.200620 \log(H-4.5)$

where V = Volume in cubic feet.

D = Diameter at breast height (inches).

H = Total height in feet.

Based on group averages, the coefficent of multiple correlation = .9990.

TABLE 24. -- *Cubic volume to a 4 inch top diameter inside bark**

(*PEELED WOOD*)

Diameter breast high	Total height of trees in feet										Basis (trees)
	40	50	60	70	80	90	100	110	120	130	
5	.50	.57	.67	.79							6
6	1.02	1.58	2.30	3.09	3.80						9
7	1.64	2.85	4.06	5.36	6.65	7.89					12
8		3.84	5.51	7.15	8.54	10.3					5
9		5.03	7.15	9.11	11.1	13.1	15.2				10
10			8.78	11.2	13.7	16.3	18.9	21.3			8
11			10.7	13.6	16.5	19.8	23.1	25.7			10
12			12.7	16.1	19.8	23.4	27.2	30.6			14
13				19.0	23.3	27.7	31.8	36.2	40.2		5
14				22.0	27.1	31.7	36.7	41.7	46.3		5
15				25.2	30.9	36.2	41.8	47.5	53.3		7
16					34.7	41.4	47.6	54.2	60.6		6
17					39.0	46.0	53.6	60.6	68.3		8
18						51.5	59.8	68.0	75.9		7
19						57.6	66.7	75.2	84.5		7
20						63.5	72.7	83.8	92.9		3
21						69.5	80.2	91.6	103		5
22						76.1	88.9	98.4	112		2
23						83.1	96.2	109	122	136	3
24						90.5	103	117	131	145	1
25							113	127	143	156	3
26							121	137	154	168	2
27							131	147	164	182	2
Basis (trees)	2	9	15	16	19	26	24	25	3	1	140

* Volumes include the peeled volume of the main stem to a 4 inch top inside bark, and exclude limbs.

Block indicates extent of basic data.

Values computed by subtracting from the computed values of the volume inside bark of the entire stem the volume inside bark from the tip to the 4 inch diameter inside bark as computed from the equation:

i.b. volume (4 inch i.b. to tip) = .16461062 D - .0062073 H + .63829666.

Standard error of estimate of volume i.b. (4 inch i.b. to tip) = .689425 cubic foot.

TABLE 25. -- *Board-foot volume (International 1/8 inch) rule to a 5 inch top diameter*

Diameter breast high	Total height of trees in feet									Basis (trees)
	50	60	70	80	90	100	110	120	130	
										6
7	9	12	15	18	22					12
8	16	22	28	34	41					5
9	24	32	40	50	61	72				10
10		42	54	68	83	100				8
11		55	70	88	109	129	150			10
12			86	108	133	159	185			15
13			104	131	162	193	226	259		10
14			122	156	192	229	266	305		9
15				181	223	264	308	358		12
16				205	252	298	350	410		9
17				230	282	336	396	458		8
18					317	378	443	507		10
19					350	419	486	557		6
20					379	456	525	606		3
21					410	491	570	664		5
22					445	540	610	722		2
23					480	560	660	778	894	3
24					500	600	710	836	956	1
25						650	770	890	1,020	3
26						690	805	950	1,080	2
27						730	850	1,000	1,140	2
Basis (trees)	4	13	16	21	30	33	30	3	1	151

Block indicates extent of basic data.

Values computed from the equation: $\log (V-2.85) = 1.2495 \log (D-5) + 1.6357 \log (H-4.5) - 2.0258$

where V = Volume in board feet (International log rule)
D = Diameter at breast height (inside bark) in inches
H = Total height of tree in feet.

Standard error of estimate in logarithms = .1211.

Coefficient of multiple correlation = .9845.

TABLE 26. -- *Board-foot volume, Scribner rule, to an 8 inch top diameter*

Diameter breast high	Total height of trees in feet								Basis (trees)
	60	70	80	90	100	110	120	130	
9	8	8	8	9					5
10	15	19	23	26	29	33			8
11	28	34	42	49	57	66			10
12		52	63	73	86	98			15
13		69	84	99	117	132	151		10
14		88	107	126	150	175	202		9
15		106	129	155	187	219	250		12
16			155	188	225	262	299		9
17			184	221	264	304	345		8
18			210	254	300	344	390		10
19			239	285	340	386	440		6
20				319	377	431	492		3
21				352	417	480	550		5
22				389	455	530	620		2
23				430	500	590	680	775	3
24				460	550	640	740	850	1
25					595	700	800	905	3
26					640	760	870	935	2
27					695	800	920	1,050	2
Basis (trees)	1	7	18	30	33	30	3	1	123

Block indicates extent of basic data.

Computed from the equation:

$$\log. (V-6.67) = 1.2431 \log. (D-8) + 1.5465 \log (H-4.5) - 1.7886$$

where V = Volume in board-feet (Scribner log rule)
D = Diameter at breast height (inside bark) in inches
H = Total height of tree in feet

Standard error of estimate in logarithms = .1387

Coefficient of multiple correlation = .9717

APPENDIX

B

TECHNIC OF GROWTH AND YIELD TABLE CONSTRUCTION

APPENDIX B

TECHNIC OF GROWTH AND YIELD TABLE CONSTRUCTION

Field Methods

The methods employed in the field work of this study were essentially those proposed in 1926 by a committee of the Society of American Foresters.[6] Basic data consist of measurements taken on 99 permanently located sample plots whose boundaries were located with a staff compass and steel tape. In general, rectangular plots were established, but where this was impracticable plots of varying shape were laid out. Acute angles were avoided. Plot sizes ranged from one-fiftieth to one-half acre, the small-sized plots having been established only in young stands where trees were small and numerous. In general,one-half acre plots were established in all stands over 50 years old. The stand-age of each plot was determined from increment cores extracted at breast height from dominant red gum trees. Throughout these yield tables, breast height age has been used instead of total age because seedlings and sprouts require different periods of time to reach breast height, and because breast height age is a more generally useful figure.

Yield plot field procedure included a careful examination of each merchantable tree (as defined on page 7). Its merchantable length was determined and divided into logs; the grade, diameter at the small end, and length of each log was estimated. From these estimates,the volume of each log was calculated by the Scribner and Doyle log rules. The aggregate merchantable volume, both total and by log grades, was computed for each plot,raised to an acre basis, and correlated with age and site. Table 1 on page 10 shows the average volumes for the various ages and sites.

Since frequent use of an Abney level maintained a satisfactory standard of accuracy in estimating heights and log lengths, the accuracy of these volume estimates depended largely upon the precision of the field mens' ocular estimates of top-of log diameters inside bark. A test of their accuracy was made through a set of taper curves constructed from volume table measurements taken on nearly 150 felled, second-growth red gum trees. Diameter estimates from these taper curves, when compared with corresponding diameter measurements on the felled trees, showed an aggregate error of -.32 percent; that is, the sum of the actual diameters was lower than the corresponding curved diameters by .32 percent. Top-of-log diameters estimated for trees on the sample plots were compared with corresponding diameters read from taper curves for trees of the given diameter at breast height. Approximately 3,670 such comparisons showed an aggregate error of -.23 percent; that is, the sum of the field estimates minus the sum of the taper curve estimates divided by the sum of the taper curve estimates equals -.0023 or -.23 percent. Individual deviations were independent of the height above the stump and of the diameter at breast height. These tests show no indication of bias or serious error in the ocular estimates.

6/"Methods of preparing volume and yield tables," Report of the Committee on standardization of volume and yield tables, Journal of Forestry,24: 653-666.

Method of Constructing Site-Index Curves [7]

In the construction of site index curves, it is generally assumed that the site index of a stand, as measured by the height of the average dominant tree at 50 years, is independent of the age of the stand. It is also assumed that the distribution of plots by site within an age class is independent of the age. Thus, a stand of given site index at one age will have the same site index at any other age, and the probability of the occurrence of a stand of a given site index will be the same at all ages.

The construction of site index curves necessitates relating three interdependent stand statistics, or statistics applying to groups of stands, to age. These are the height of the average dominant tree, the standard deviation of these heights around the curve of the average for an age class, and the coefficient of variation. The interdependence of these statistics is shown by the definition of the coefficient of variation. That is:

$$V = \frac{\sigma\ (100)}{M}$$

where

V = coefficient of variation,

σ = standard deviation of heights, and

M = curved value of height for the average site at a

given age.

Hence, an independently drawn curve of the coefficient of variation over age can be compared with one derived from the curves of height and the standard deviation of heights over age and thereby provide a check on the forms of all three.

The steps followed in constructing site index curves are:

(1) Plot the height of the average dominant tree over the stand age. A curve through these points represents the relation of height to age for the average of the site indexes.

(2) Compute the standard deviation of heights around this curve, for 10-year (or other interval) age classes, and plot aver age. A curve drawn through these points represents the trend in the variation among heights of dominant trees within an age class as the age class changes.

(3) Compute the coefficient of variation of each of these age classes, plot over age, and fit a curve to these points. This curve provides a check for curves drawn under (1) and (2).

7/ For a more complete description of the yield and stand table construction methods used here, see "The Construction of Normal-Yield and Stand Tables for Even-Aged Timber Stands," by Osborne and Schumacher, in Journal of Agricultural Research, Vol. 51 (6): 547-564.

(4) For the reference age - 50 years - divide the deviation of each even 10-foot site index from the average site index by the curved standard deviation at 50 years to obtain the deviation in standard units of that site from the average. The deviation of each site from the curve of the average site at any given age is the product of the deviation of that site from the average in standard units at 50 years by the curved standard deviation at the given age.

(5) Derive curves for each 10-foot site index class, and from these curves assign a site index to each plot.

The distribution of plots by age and site classes is shown in the following table.

TABLE 27. -- *Distribution of plots by age and site-index class*

Breast height age class	Site-index in feet					All plots
	70-79	80-89	90-99	100-109	110-119	
Years	*Number of plots*					
5.0 - 15.0		2	5	3	1	11
15.1 - 25.0	1	3	1	11	1	17
25.1 - 35.0	1	1	3	6		11
35.1 - 45.0		2	4	4	1	11
45.1 - 55.0	2	2	6	7	3	20
55.1 - 65.0	2		3	6	2	13
65.1 - 75.0			3	1	1	5
75.1 - 85.0	1		3	1	1	6
85.1 - 95.0				2		2
95.1 - 105.0			2	1		3
Total	7	10	30	42	10	99

Method of Constructing Curves of Total Stand Basal Area, Number of Trees, Average Diameter, and Cubic Volume

Basal area, number of trees, and cubic foot volume per acre, as well as average diameter, were correlated with stand age and the assigned site index. For each of these four measures of yield, the first three steps outlined on page xviii were followed. As with height, the curve of each of these measures of yield for a given site deviates from the curve for the average site by a certain number of units of the standard deviation. To obtain this relationship for cubic volumes, for example, the deviation of each plot yield from the yield for the average site at the plot age was divided by the standard deviation of yield at that age. These ratios, or standard units, were then plotted over site index and a curve was fitted to the points. The deviation of the yield for any site and age from the curve of yield for the average site at that age is the product of the value read from the curve of the standard deviation at that age by the value read from the curve of standard units for that site. Through deviations computed by this procedure, for selected ages and sites, curves of yield of basal area, number of trees, and average diameter were also constructed for 10-foot intervals of site index.

Method of Construction of Partial Stand Yield Curves

Studies in fully stocked, even-aged forest stands in a given timber type have shown that stands of the same average diameter have diameter distributions that are closely alike. Therefore, the portion of the stand above a given diameter should be about the same in stands of equal average diameter, and the ratio of partial to entire stand values should be correlated with average diameter. These ratios have not been proved to be completely independent of age and site, but have been found satisfactorily so in practice.

The procedure followed in the construction of the partial stand yield curves is that of correlating the ratio of partial to entire stand values with average diameter. As an example of this procedure, the basal area of trees in and above the 7-inch diameter class was summed for all plots of the same average diameter; this sum was divided by the sum of the basal areas of all trees on the same plots, and the resultant ratios were plotted over the average diameter of these plots. A curve through these values provided a ratio corresponding to any average diameter and therefore to any age and site. The basal area in trees in and above 7-inch diameter class could then be obtained for any age and site by multiplying the total basal area by the ratio of partial to total basal area for that age and site. Curves of basal area in and above the 7-inch diameter class were constructed for each 10-foot site class. A similar procedure was followed in deriving all partial stand curves.

Method of Constructing Stand Tables

The general plan followed was that of fitting to diameter distributions of stands of the same average diameter a curve of the type

$$y = \frac{100}{1 + e^{a + b x + c x^2 + d x^3}}$$

where

y is the cumulative frequency in percent, through a diameter class,

x is the diameter breast-high in inches,

e is the base of Napierian logarithms, and

a, b, c, and d are constants calculated by the method of least squares for the average of all plots having the same average diameter.

The constants a, b, c, and d were then adjusted, and cumulative frequency curves drawn for successive inches of average diameter. By interpolation, these curves were converted to curves for stands of successive inches of average diameter, as computed from the basal area of the tree of average basal area.

Cross Check Between of Basal Area, Average Diameter, and Number of Trees

For a given plot, the total basal area equals the product of the number and the average tree basal area. Expressed algebraically, this relation is

$$B = (n\ b)$$

where B = the total basal area of a plot,

n = the number of trees on the plot,

and b = the average basal area per tree.

But $b = K\ \overline{D^2}$*

where $\overline{D^2}$ = the average of the squared diameters

K = a constant.

Because this relationship holds for an individual plot, it has been assumed that it will be true for the average of all plots within an age class. The practice of many investigators has been to derive the curve of $\sqrt{\overline{D^2}}$ (i. e., the diameter of the tree of average basal area) from the ratio of the curves of B and n. That this has been in error and has been found to be an unwarrented cross-check, is demonstrated in the following:

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*A dash over any symbol means that it represents an average value.

For a given age class, if the average total basal area per plot equals the product of average number of trees and the average basal area of the average tree, the average of a number of cross products equals the product of the averages, or

$$\bar{B} = \bar{n}\ \bar{b}$$

for all plots within the age class.

$$B_1 = n_1 \cdot b_1$$

$$B_2 = n_2\ b_2$$

" " "

" " "

' ' '

$$B_N = n_N\ b_N$$

$$\frac{\sum B_i}{N} = \frac{\sum n_i\ b_i}{N} \qquad i = 1, 2, \cdots N.$$

$$\text{But} \sum n_i\ b_i = \sum (n_i - \bar{n})\ (b_i - \bar{b}) + N\ \bar{n}\ \bar{b}$$

$$\text{and } \bar{B} = \frac{\sum (n_i - \bar{n})\ (b_i - \bar{b})}{N} + \bar{n}\ \bar{b} \qquad (1)$$

$$\text{or} \quad \bar{B} = r_{nb}\,\sigma_n\,\sigma_b + \bar{n}\ \bar{b} \qquad (2),$$

From equation (2)

$$B \neq \bar{n}\ \bar{b}$$

unless either

r_{nb}, σ_n, or σ_b is zero.

This can occur only if n or b is constant or if they are uncorrelated. In all investigations, n and b have been found correlated with site. Hence, they must be correlated with each other.

In the present paper, therefore, independent curves have been drawn as the best approximations of the relationships of B, n, and $\bar{D}$ to age and site, without regard to cross-checking.

ERRATA

Appendix

Page iv - Table 8 - Read title - "Cubic-foot volume per acre of all trees 0.6 inch d.b.h. and larger."

Page iv - Table 9 - Footnote 2 - read - "average percentage," in place of "average percentages."

Page xix - Table 27 - under Breast height age class, read "5.1 - 15.0" instead of "5.0 - 15.0"

Page xx - Line 4 - read - "As with height, the curve of each of these *four* measures of yield.....".

Page xxi - Side heading in middle of page - read - "Cross Check Between *Curves* of Basal Area....", and immediately below, "For a given plot, the total basal area equals the product of the number *of trees* and the average tree basal area."

(Note: Words in italics are those omitted from original text.)